INAUGURAL LECTURE TO THE SERBIAN ACADEMY OF SCIENCES AND ARTS

April 22, 2025

From Hooke's "Hanging Chain" and Milankovitch's "Druckkurve" to a Variational Formulation: The Adventure of the Thrust Line of Masonry Arches

Dr. Nicos Makris

Professor in Civil Engineering

Southern Methodist University, Dallas, Texas

Abstract

At the dawn of the 20th century, the Serbian engineer and astronomer, Milutin Milankovitch presented a remarkable formulation for the thrust line of masonry arches that do not sustain tension and by taking radial cuts, he published for the first time the correct and complete solution for the theoretical minimum thickness, t of a monolithic semicircular arch with radius, R capable to carry its own weight.

In this lecture, we first explain that Milankovitch's result, t/R=0.1075 is not unique and that it depends on the stereotomy exercised. The adoption of other than radial stereotomies yields neighboring thrust lines and different, slightly higher values for the minimum thickness than the value computed by Milankovitch. This result, that serves safety, has been obtained with a geometric and a variational formulation. The Milankovitch minimum thrust line derived with radial stereotomy, and the other neighboring minimum thrust lines derived with arbitrary stereotomy are distinguishable physically admissible thrust lines which do not coincide with R. Hooke's catenary that meets the boundaries of the arch at the three extreme points. The lecture concludes that the catenary (the hanging chain), which has been used widely since the period of the enlightenment to assess the stability of masonry arches, is not a physically admissible thrust line. The minimum thickness of a semicircular arch that is needed to accommodate the catenary curve is t/R=0.1117—a value that is even higher than the larger minimum thickness value, t/R=0.1095 that results from the most conservative physically admissible thrust line that is computed after adopting a vertical stereotomy